Flow Modelling and Velocity Distribution in Small Irrigation Canals

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Abstract: *Accurate metering of agricultural water is becoming increasingly important world wide. Surprisingly, the dynamics of water in small irrigation canals are not well understood and this limits the accuracy of flow measurements in these channels. To help improve the accuracy of flow data, we performed detailed measurements of the velocity distribution for a variety of flow conditions in a several small irrigation canals. The data collected represent one of the most detailed velocity distribution studies ever undertaken in field conditions for this type of channel. The data were compared with different theoretical velocity distribution models. The results of these comparisons will be presented, along with conclusions about which models are most appropriate for predicting the velocity distribution in these channels. The goal of this project is to support the development of new flow sensors for small irrigation canals that provide improved measurement accuracy.*

Keywords: *irrigation, flow, measurement, water, velocity, modelling*

1. INTRODUCTION

In much of the world, the majority of agricultural water is delivered through small open irrigation canals. Water measurement is extremely important – the water resource cannot be managed well unless it is measured and controlled throughout the complete web of water delivery and recirculation systems. Based on discussions with water managers and industry leaders, we found a significant gap in available instrumentation for channels with depths ranging from 0.05 to 1.5 m, and widths from 0.3 to 5.0 m. There are tens of thousands of these channels around the world; in one part of the western United States alone, one survey identified 35,000 such sites (ITRC 1996). Other regions of the world with extensive networks of these canals include Australia, China, India, and southern Europe.

Existing flow measurement technology is often based on water level using some type of control structure (a rated gate, weir or flume), or may use velocity plus water level measurement via electro-magnetic (EM) or mechanical means. Control structures typically require a loss of head, which for many canal systems is not feasible. Existing water velocity instruments often rely on a measurement at a single point within the channel that may not be representative of the total cross section. As complex flow conditions exist even in these small channels, detailed measurements of the velocity distribution, in addition to water level, will yield a more robust flow measurement across a wider range of channel conditions.

To accurately measure flow, it is necessary to understand the flow conditions that exist within these small channels. We performed detailed measurements of the velocity distribution in several different channels to help provide this understanding. These measurements were then compared to different velocity distribution models to see which models most accurately reproduce the flow conditions seen at these sites. The results of these comparisons are presented here. From these comparisons, we conclude which models most accurately represented the flow conditions that we observed.

2. VELOCITY DISTRIBUTION MEASUREMENTS

We were unable to locate any existing detailed velocity distribution data for these channel types, so we performed measurements at a number of sites in the western United States. The measurements were performed in cooperation between SonTek/YSI and the Irrigation Training and Research Center (ITRC) of California Polytechnic State University, San Luis Obispo, California.

2.1. Data Collection Procedure

All velocity measurements were made with a SonTek/YSI FlowTracker®, a high precision single point velocity sensor. A preliminary site was sampled with a larger measurement density and duration than would be practical for measurements at multiple sites. Results from this site were used to develop field data collection procedures. Data for each field site includes a detailed survey of channel geometry and velocity measurements at 10 or more locations across the width of the channel and 1-6 different depths for each location. Each velocity measurement lasted 40 seconds.

Sites were generally selected at a transition point within a channel; this might be a control gate, a bend, or a change in channel geometry. Measurement cross sections were then located at different distances moving downstream from the transition point, providing data to examine the distance required to reach a well defined flow distribution. Depending upon the channel geometry and other logistical details, a single site required about 4 hours for a team of two people to complete the required field work. Measurements were coordinated with local irrigation managers to ensure steady flow conditions during the period of each measurement.

A temporary walkway was placed across the channel so that measurements could be made without requiring the operator to enter the water, which would significantly alter the velocity distribution. When practical, the same site was sampled at different flow conditions; in practice, this was only possible at a limited number of sites since flow conditions were determined by the water requirements at any given time and could not normally be adjusted to suit our measurement needs.

Figure 1 Example Measurement Site with Walkways for Two Cross Sections

2.2. Example Velocity Distribution Data

Figure 2 shows an example of velocity distribution data where a total of 57 velocity measurements were made at multiple points along 12 different verticals; the location of each measurement is shown with an asterisk (*). The velocity distribution in the canal is shown using a contour plot, with different colours representing different velocities as shown in the scale to the right of the figure. The site in Figure 2 represents one of the more uniform and predictable velocity distributions seen in our data. The velocities are skewed slightly towards the right side of the channel, but in general it shows decreasing velocities that would be expected as you approach the bottom and sides of the channel. It was interesting to discover the degree of variation seen in these measurements, even in seemingly perfect measurement sites. Figure 3 shows the distribution at a cross section in the middle of a long, straight, uniform, concrete lined canal that had been cleaned immediately prior to these measurements. The nearest change in the canal was more than 100 m away from this site. This illustrates that even at seemingly perfect measurement sites, detailed measurements of the velocity distribution are needed for an accurate flow calculation. For sites with large amounts of sediment on the bottom or other irregularities, the velocity distribution was often highly unpredictable.

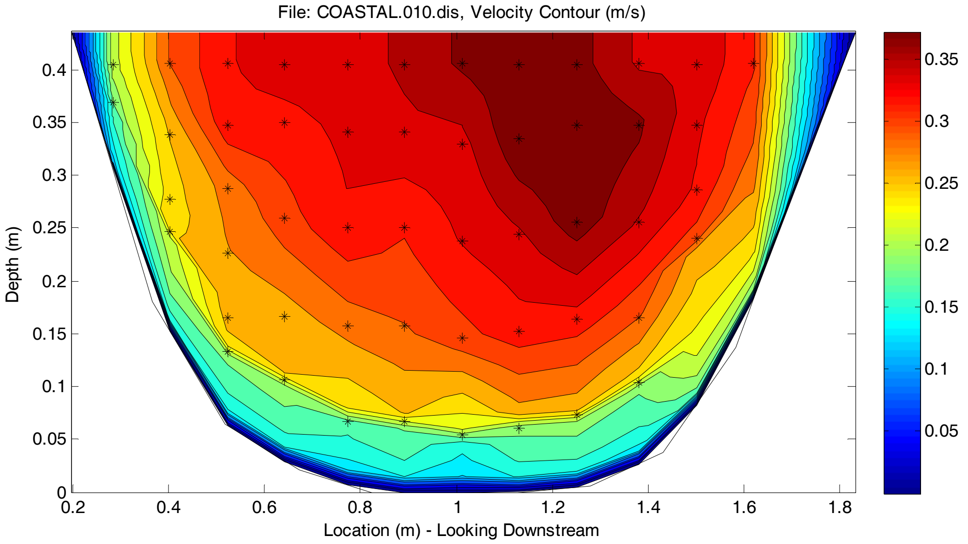

Figure 2 Example Velocity Data

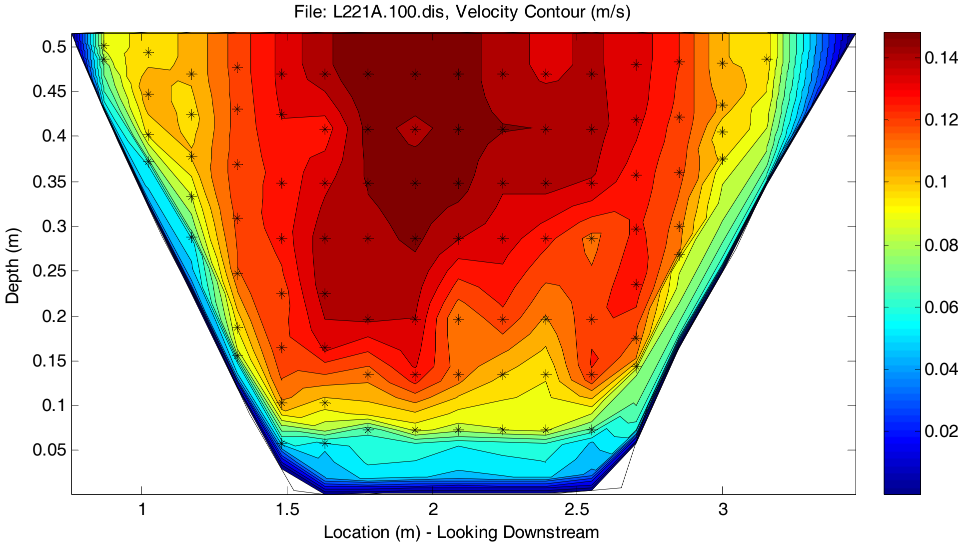

Figure 3 Example of Irregular Velocity Distribution

3. FLOW MODEL COMPARISON

The objective of the flow modelling in this project was to assist in the development of a new sensor to measure flow in small canals. The new sensor will use pulsed acoustic Doppler profiling technology to measure the velocity in some portion of the canal. Acoustic Doppler systems can be split into two categories: continuous wave and pulsed. As the name suggests, continuous wave systems transmit and receive continuously, measuring the velocity of water along the entire beam path at the same time. It is not possible to distinguish the exact location of the measurement nor is it possible to know the spatial distribution of the velocity measurement along the beam. Pulsed Doppler systems transmit a short acoustic pulse and measure the response versus time, thereby measuring a profile of velocity along the path of the acoustic beam. A system designed for small canals can achieve a resolution of the spatial distribution of velocity to the order of a few centimetres. A system with multiple beams oriented in different directions can measure the velocity along each beam, thus giving the velocity distribution in different portions of the canal.

The goal of our flow modelling is to determine the number and orientation of acoustic beams needed so that the total flow in the channel can be extrapolated from the data measured by those beams. In particular, we wanted to determine the accuracy of the flow estimate for a given environment and beam configuration. Several models were evaluated and compared to data collected in the field. Given space limitations, only a brief description of each model is given in the sections that follow.

3.1. Power Law

The power law is one of the most widely used models for open channel flow. Eq. (1) below is a common expression of the power law velocity distribution (Chen, 1991).

$$\frac{u}{u*} = a\left(\frac{y}{y'}\right)^m \qquad (1)$$

where *u is* velocity at any point ; *u** is boundary shear velocity; *a* is a constant; *y* is the distance to the boundary (for velocity point *u*); *y'* is the characteristic length for zero velocity isovel; and *m* is a constant, typically in the range 1/2 to 1/10 (1/6 is a common value).

We are using the power law to fit and extrapolate from measured velocity data, so we do not attempt to derive values of *y'* and *u** from boundary conditions. Thus we can incorporate these into the constant a_1 giving the simpler expression of Eq. (2).

$$u = a_1 y^m \qquad (2)$$

The power law was developed for the ideal cases of a wide channel of constant depth or an axisymmetric conduit; boundary distance is the vertical height above the bottom for the former and the radial distance from the boundary for the latter. For a typical irrigation canal, this value cannot be accurately used as we approach the walls of the channel. We have instead used the shortest normal (perpendicular) distance to the boundary for *y,* which is consistent with boundary-layer theory.

The exponent *m* is a key parameter in applying the power law, as it is the primary driver for the shape of the velocity distribution. The value of *m* is influenced by a number of factors, perhaps the two most important being boundary roughness and turbulence. We have treated these as constant for any given cross section in a given flow conditions; thus a cross section will have a single m value to represent the entire cross section, though *m* might vary with time.

When applying the power law to velocity data, we have tried a number of variations to see which would yield the best results.

- Computing the optimum value of *m* based on all velocity data in the cross section.
- Computing the optimum value of *m* based only on the vertical velocity profile in the centre of the channel. This would be comparable to having an instrument that measures the vertical profile of velocity in the middle of the channel.
- Computing a single value of the constant *a* for the entire cross section.
- Allowing the value of a to vary across the width of the channel. This would be comparable to having an instrument that measures the horizontal variation of velocity across the channel.

Another common flow theory is the logarithmic law (Chen, 1991). Both log law and power law are members of a family of functions that can simultaneously satisfy the partial differential equations for the ‘inner law’ and the ‘outer law’ in fluid mechanics. From a fluid mechanics and mathematical perspective, the two theories are equivalent; we selected the power law because it is easier to apply.

3.2. Chiu’s Maximum Entropy Method

In recent years, a probabilistic approach to modelling open channel flow has gained increasing acceptance; it is sometimes called the maximum entropy method (Chiu, 1989, Chiu & Hsu 2006). This method predicts a constant relationship between the mean and maximum velocity at any given cross section, and thus provides a theoretical underpinning for the widely used index velocity method (Morlock et al, 2001). The method is very attractive as it simplifies the monitoring of discharge by reducing the number of velocity measurements required.

The maximum entropy method can also be used to calculate the velocity distribution in a channel. Predicting velocity distribution in a channel requires coupling the entropy-based equation for velocity (Eq. 3) with an equation describing the shape of isovel curves based on the geometry (Eq. 4).

$$u = \frac{u_{max}}{M} ln\left[1 + (e^M - 1)\frac{\xi}{\xi_{Max}}\right] \tag{3}$$

$$\xi = Y(1 - Z)^{\beta_i} \exp(\beta_i Z - Y + 1) \tag{4}$$

where $u_{Max}$is the maximum velocity in the cross section; M defines the ratio of mean to maximum velocity $\left(\frac{\bar{u}}{u_{max}} = \frac{e^M}{e^M - 1} - \frac{1}{M}\right)$ (typically in the range 1 to 10); ξ is an index variable representing velocity on an isovel; Y and Z are dimensionless coordinates in vertical and lateral direction, respectively; β_i are coefficients to define horizontal distribution (typically in the range 1 to 10), i=1 for left of Y_{Axis} and i=2 for right of Y_{Axis}; Y_{Axis} is the location of the maximum velocity across the width of the channel. Hence, to define the velocity distribution, we have four independent parameters (M, Y_{Axis}, β_1, and β_2).

With detailed velocity measurements throughout the cross section, we can reasonably assume that we have measured the maximum velocity and so we can know the location Y_{Axis} directly. The parameter M was determined two ways.

- M can be calculated from the ratio of the mean to maximum velocity in the channel, which can be directly calculated from our velocity measurements.
- The vertical velocity profile at the Y_{Axis} can be predicted based on the value of M and the measured maximum velocity. Thus we can determine a value of M by performing a best fit of the vertical velocity profile as measured at the Y_{Axis}.

The remaining parameters, β_1 and β_2, were fit to the measured velocity data using a least-squares approach once we determined values for Y_{Axis} and M.

3.3. Maghrebi and Rahimpour's Approach

One significant limitation of many flow models (including the power law and logarithmic law) is that they are developed for an ideal boundary condition of a wide, shallow stream with a flat bottom. This is a problem in narrower channels where the exact bottom contour and wall effects must be taken into account. An interesting model has been recently proposed that provides a method for accounting for bottom and wall effects by integrating any other model of open channel flow to account for an arbitrarily shaped channel boundary (Maghrebi & Rahimpour, 2005, Maghrebi & Rahimpour, 2006). The basic idea of this model is that every part of the boundary has an effect on the velocity at each point within the boundary. So given a model that describes the effect of the boundary on velocity, if you integrate this model over the true boundary shape you can describe the flow pattern in any arbitrarily shaped boundary.

Although we found this concept extremely intriguing, we encountered a problem that prevented its practical use. Most models were developed not for the influence of a single point on the boundary but rather for the influence of an assumed boundary shape. The simplest way to illustrate this is to look at the basic power law relation shown in Eq. (2). This equation says that the velocity at some distance y from the boundary is proportional to that distance raised to the power m. However, if we integrate Eq. (2) over an arbitrary boundary shape, the exponent changes, thereby dramatically changing the basic shape of the velocity distribution. For example, for the ideal case of the wide flat channel for which the power law was developed, the exponent in Eq. 2 approaches unity, resulting in a linear velocity profile. We attempted various modifications to this theory to avoid this problem, but were unable to find an effective method. It is our hope that additional work on this concept can overcome these problems, as seems to hold promise as an effective tool to predict velocity in any arbitrarily shaped channel.

3.4. Evaluation Criteria

We determined a best fit of each theoretical distribution for a particular channel, using some or all of the velocity data collected (depending on the particular theory and implementation). Using these best-fit parameters, we then calculated the predicted velocity at the location of each of our velocity measurements in the cross section. We then calculated the root mean square (RMS) of the difference between the predicted and measured velocity values. The RMS error was converted to a percentage dividing by the mean channel velocity for that cross section; mean channel velocity is discharge divided by cross sectional area.

As a second evaluation criterion, we used the same routine to estimate velocity data at each measured location. We then compared the total channel discharge using the measured velocity data, and compared this to discharge calculated using the theoretically predicted velocity data. This "discharge error" is particularly relevant for our study as our final goal is to assist in the development of a new sensor for monitoring flow in this type of channel.

4. COMPARISON RESULTS

Fifteen cross sectional measurements were selected for detailed analysis. These all have a reasonably well developed velocity distribution, and represent a good variety of small irrigation canals. Table 1 below provides basic details for each cross section. If the same location was measured at different flow conditions, the rows from that location have been highlighted in the same colour.

Table 1 Description of Measurement Cross Sections

Site	Lining	Q (m^3/s)	Depth (m)	Mean Vel (m/s)	Comments
1	Concrete	0.22	0.41	0.36	Clean channel
2	Natural	0.62	0.41	0.47	On gradual bend, stable bed
3	Concrete	0.16	0.44	0.30	Clean channel
4a	Concrete	0.11	0.61	0.09	Heavy bed sediment, higher flow rate
4b	Concrete	0.12	0.32	0.24	Clean channel, higher flow rate
4c	Concrete	0.03	0.40	0.05	Heavy bed sediment, lower flow rate
5a	Concrete	0.12	0.71	0.07	Heavy bed sediment, higher flow rate
5b	Concrete	0.11	0.52	0.12	Clean channel, higher flow rate
5c	Concrete	0.03	0.48	0.03	Heavy bed sediment, lower flow rate
6a	Concrete	0.11	0.69	0.12	Heavy bed sediment
6b	Concrete	0.12	0.69	0.17	Clean channel
7	Natural	0.05	0.30	0.09	Stable bed
8	Concrete	0.55	0.74	0.39	Clean channel
9	Concrete	0.40	0.65	0.34	Clean channel
10	Concrete	0.48	0.69	0.56	Clean channel

The following models were evaluated for how well they matched measured data.

A. Power Law #1: *m* value from entire cross section, single *a* value for entire cross section.
B. Power Law #2: *m* value from entire cross section, vary *a* across width of channel.
C. Power Law #3: *m* value from centre of channel, single *a* value for entire cross section.
D. Power Law #4: *m* value from centre of channel, vary *a* across width of channel.
E. Maximum Entropy #1: Y_{Axis} based on location of maximum velocity, M from mean and maximum velocity, best fit β_1 and β_2 parameters.
F. Maximum Entropy #2: Y_{Axis} based on location of maximum velocity, M by best fitting the vertical velocity profile at the Y_{Axis}, best fit β_1 and β_2 parameters.

Table 2 shows the RMS velocity error for all models at each cross section, as a percentage of the mean velocity at that cross section. Table 3 shows the percent error in total discharge for all models at each cross section. The error is determined by comparing discharge calculated using the measured velocity points to discharge calculated using the predicted values, as a percentage of discharge from measured velocity data.

Analysis of table 2 indicates that the models all performed equally when considering the RMS error of the individual velocity points. While some measurements showed relatively small errors (< 10%) among all methods, median RMS errors were around 20% with some measurements showing RMS errors in the velocity larger than 50%. All of the velocity distribution models imply a well-behaved relation (e.g., continuous function and gradient, usually monotonic); in reality velocity distributions often are not mathematically well behaved (e.g., Fg. 3), resulting in large errors between the mathematical function and the measured velocities. It is worth noting that the measurements with the largest RMS error (4a and 4c) were located at the same cross section as one of the measurements with the lowest RMS error (4b). However, measurement 4b was after the channel was cleaned. Furthermore, all of the measurements with RMS errors less than 10% are for clean concrete channels. This implies that prediction of the velocity distribution in the cross section is affected by the condition of the channel.

Table 2 RMS Percent Error of Velocity Data

Site	Model A	Model B	Model C	Model D	Model E	Model F	Median
1	6.0%	7.0%	6.2%	6.9%	9.7%	9.6%	7.0%
2	24.7%	22.6%	32.3%	18.1%	15.9%	17.6%	20.4%
3	6.1%	5.0%	6.5%	4.9%	10.9%	10.7%	6.3%
4a	38.7%	33.1%	51.3%	33.8%	49.6%	48.8%	43.8%
4b	6.0%	6.2%	6.3%	6.2%	14.2%	12.9%	6.3%
4c	50.2%	62.2%	55.3%	60.3%	94.6%	93.6%	61.3%
5a	19.1%	19.7%	26.3%	26.5%	29.4%	29.4%	26.4%
5b	9.0%	10.2%	9.3%	10.4%	14.2%	14.2%	10.3%
5c	23.4%	33.6%	29.9%	32.3%	35.4%	35.4%	33.0%
6a	20.9%	14.2%	21.8%	14.9%	21.0%	21.6%	21.0%
6b	16.7%	14.5%	20.5%	14.4%	16.7%	15.1%	15.9%
7	17.9%	18.3%	19.1%	18.3%	14.6%	14.9%	18.1%
8	19.7%	14.3%	24.7%	17.8%	17.5%	16.9%	17.7%
9	17.5%	14.8%	20.2%	14.5%	22.9%	22.6%	18.9%
10	8.4%	8.9%	9.4%	8.2%	7.5%	8.0%	8.3%
Mean	**19.0%**	**19.0%**	**22.6%**	**19.2%**	**24.9%**	**24.8%**	
Median	**17.9%**	**14.5%**	**20.5%**	**14.9%**	**16.7%**	**16.9%**	

Table 3 Percent Error in Discharge Calculation

Site	Model A	Model B	Model C	Model D	Model E	Model F	Median
1	-0.4%	2.0%	-0.1%	1.5%	5.0%	4.6%	1.8%
2	-3.2%	9.3%	15.1%	4.0%	3.8%	5.5%	4.8%
3	-0.4%	-1.4%	-1.6%	-1.3%	8.0%	7.3%	-0.9%
4a	9.7%	-0.7%	-22.5%	1.7%	27.4%	28.0%	5.7%
4b	0.2%	-0.4%	1.6%	-0.5%	8.8%	7.7%	0.9%
4c	6.2%	-23.8%	-16.5%	-23.4%	7.9%	29.5%	-5.2%
5a	3.1%	9.5%	-12.1%	16.6%	18.6%	20.6%	13.1%
5b	3.0%	3.3%	2.2%	3.5%	7.0%	7.4%	3.4%
5c	-1.3%	-16.1%	-11.3%	-14.7%	23.9%	19.9%	-6.3%
6a	3.0%	5.0%	2.9%	6.4%	11.0%	13.7%	5.7%
6b	-1.3%	3.6%	7.7%	1.8%	6.2%	4.7%	4.2%
7	-3.9%	-0.8%	2.4%	-1.6%	0.6%	1.0%	-0.1%
8	0.7%	1.1%	-11.1%	9.3%	8.3%	6.7%	3.9%
9	-2.9%	-5.2%	-11.9%	-4.1%	11.8%	11.5%	-3.5%
10	1.4%	-1.9%	-1.7%	-0.7%	5.6%	4.2%	0.4%
Mean	1.0%	-1.1%	-3.8%	-0.1%	10.3%	11.5%	1.9%
Median	0.2%	-0.4%	-1.6%	1.5%	8.0%	7.3%	1.8%
Mean(Abs)	2.6%	5.6%	8.1%	6.1%	10.3%	11.5%	7.1%
Median(Abs)	2.9%	3.3%	7.7%	3.5%	8.0%	7.3%	4.8%

Table 3 indicates that despite the large errors in predicting the velocity distribution, the models based on the power law provided a relatively good prediction of the discharge past the cross section, with the median of the errors on the order of 3% and the number of overestimates and underestimates nearly equal. Model B, which has more adjustable coefficients than Model A, results in larger errors than Model A. This is a result of the increased influence of profiles that do not behave as the models predict when modelling section-by-section. In contrast to the power-law models, the models based on

Chiu's maximum entropy method consistently overestimated the discharge past the cross section, with all discharge predictions overestimating the measured value. Chiu's method uses the parameter M to assign velocities to the isovels from (Eq. 4). While results from laboratory flume studies and selected measurements on larger streams showed good results, our results indicated that neither determining M from the ratio of the mean and maximum velocities nor determining M from the velocity profile at the Y_{Axis} provided a good estimate of the velocity distribution for these small channels. It is interesting to note that the two different methods of calculating M often yielded significantly different values for this key parameter. Determining M from the velocity profile along the Y_{Axis} often provided a poorer estimate of the flow in the channel than determining M from the mean and maximum velocities.

5. CONCLUSIONS

Several conceptual models of 2-D velocity distribution for open channels were examined to see which models most accurately reproduce the flow conditions seen at these sites. The model of Maghrebi & Rahimpour, while providing an intriguing approach to account for arbitrary-shaped cross sections, does not maintain the basic premise of velocity distributions even for an ideal case of a flat plane. Hence this was not examined further. Comparison of models based on the power law and Chiu's maximum entropy approach indicated that both approaches perform similarly in terms of estimating the velocity distribution with a RMS error on the order of 20%. However, examination of the discharges determined based on these models indicates that the models based on the power law provide an unbiased discharge prediction with errors on the order of 3% while models based on the entropy approach consistently overpredicted the discharge on the order of 8%.

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